

Nonlinear Micro Income Processes with Macro Shocks

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Motivation

- Large macro literature quantifying how aggregate shocks affect individual outcomes:
 - Transmission of monetary policy is uneven across households (Holm, Paul, Tischbirek, 2021).
 - Household heterogeneity influences impact of recessions (Krueger, Mitman, Perri, 2016).
 - Heterogeneous exposures shape optimal policies (Bhandari, Evans, Golosov, Sargent, 2021).
 - Large empirical literature has uncovered key facts about income risk:
 - Different degrees of persistence (Hall, Mishkin, 1982; Blundell, Pistaferri, Preston, 2008).
 - Non-Gaussian shocks (Geweke, Keane, 2000; Guvenen, Karahan, Ozcan, Song, 2021).
 - Nonlinear persistence (Arellano, Blundell, Bonhomme, 2017).
 - Business cycles (Storesletten, Telmer, Yaron, 2004; Guvenen, Ozcan, Song, 2014).
- ➔ **This paper:** Framework to integrate nonlinear income processes with aggregate shocks.

Our framework

- Nonlinear micro **income process** with macro **business cycle** state:

$$\begin{aligned}\eta_{it} &= Q_{\eta}(\eta_{i,t-1}, Z_t, Z_{t-1}, u_{it}), \\ Z_t &= Q_Z(Z_{t-1}, V_t),\end{aligned}$$

- u_{it} and V_t are micro and macro shocks.
- η_{it} and Z_t are potentially unobserved, linked to micro and macro data.

Our framework

- We recover Z_t from external aggregate time series via a factor model.
- We recover η_{it} from panel data on household income using a flexible ABB-style micro income process with Z_t as additional arguments.
- We adopt a **time series of panels** approach:
 - Collection of short panels evolving over time concurrently with the macro data.
 - In the spirit of Storesletten, Telmer, Yaron (2004).
- Tools for identification, estimation, impulse response analysis and risk quantification.

How do macro shocks and income dynamics interact?

- ① How do macro shocks impact income dynamics?
 - Nonlinear persistence, conditional skewness, exposure to aggregate shocks, etc.
- ② How do macro and micro shocks propagate over time?
 - Impulse response functions to macro and micro shocks.
- ③ How much income risk do macro and micro shocks imply?
 - Compensating-variation estimates of the cost of risk.

Summary of results

① How do macro shocks impact income dynamics?

During recessions ...

- Persistence goes up for low- η /good micro shocks, down for high- η /bad micro shocks.
- Conditional skewness at all levels of η shifts down.
- Exposure to macro risk is amplified, particularly for bad micro shocks.

② How do macro and micro shocks propagate over time?

- Short-lived responses to macro shocks + persistent responses to micro shocks.
- Distributional responses to macro shocks are U-shaped in the micro ranks.

③ How much income risk do macro and micro shocks imply?

- Large cost of business cycle risk stems from nonlinear micro impact of macro shocks.

Selected literature

- **Income dynamics (with and without business cycles):** Storesletten, Telmer, Yaron (2004), Guvenen, Ozkan, Song (2014), Arellano, Blundell, Bonhomme (2017), Guvenen, McKay, Ryan (2022), Guvenen, Pistaferri, Violante (2022), GRID project, ...
- **Estimation of heterogeneous agents models using micro data:** Arellano, Bonhomme (2017), Liu, Plagborg-Møller (2023), Fernández-Villaverde, Hurtado, Nuño (2023), ...
- **Econometrics of macro-micro data:** Tobin (1950), Chetty (1968), Maddala (1971), Hahn, Kuersteiner, Mazzocco (2020), Chang, Chen, Schorfheide (2024), Almuzara, Sancibrián (2025), ...
- **Inequality and business cycles, welfare cost of fluctuations:** Krusell, Smith (1998), Bhandari, Evans, Golosov, Sargent (2021), Lucas (1987, 2003), ...

Outline

- ① Framework
 - Model & objects of interest
 - Identification & estimation
- ② Macro shocks and nonlinear dynamics
- ③ Macro/micro impulse responses
- ④ Macro/micro risk quantification
- ⑤ Conclusion

Framework

Model & objects of interest

Model: nonlinear income process

- Persistent-transitory decomposition:

$$y_{it} = \eta_{it} + \varepsilon_{it}.$$

- Markovian persistent component **with macro state Z_t** :

$$\eta_{it} = Q_{\eta}(\eta_{i,t-1}, Z_t, Z_{t-1}, u_{it}),$$

u_{it} serially independent $U(0, 1)$ rv's, independent of $\eta_{it_0}, \varepsilon_{it}, Z_{t_0}$.

- Initial condition and transitory component:

$$\eta_{it_0} = Q_{\text{init}, t_0}(\nu_{it_0}),$$

$$\varepsilon_{it} = Q_{\varepsilon, t}(v_{it}).$$

Model: business cycle indicator

- Factor model for macro data $W = (\text{GDP}, C, I, \text{urate}, \text{hours})$:

$$W_t = \Lambda Z_t + e_t.$$

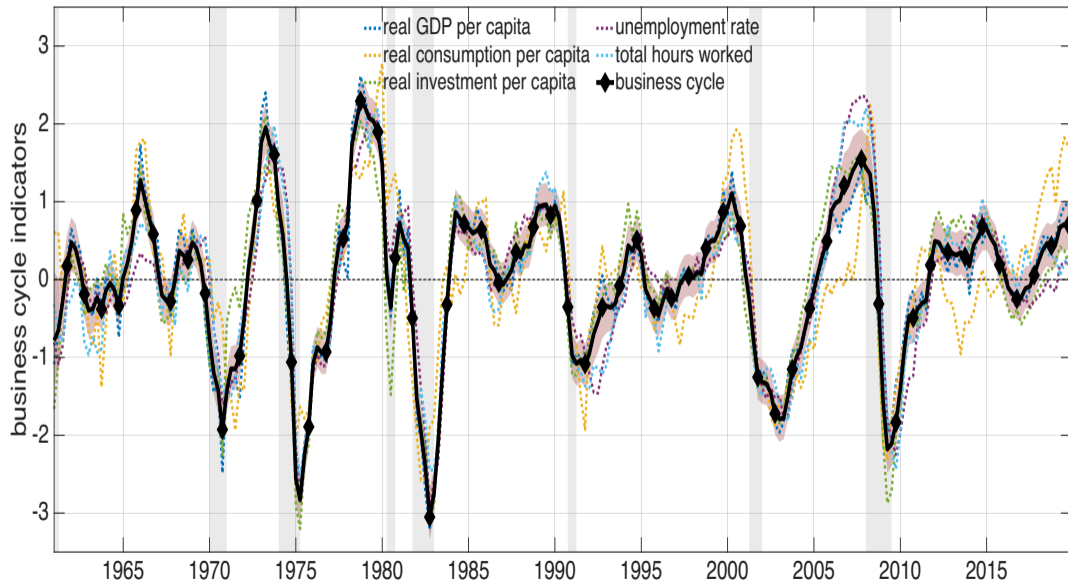
- Named-factor normalization $\implies \Lambda_{\text{GDP}} = 1$.
- Markovian business cycle state:

$$Z_t = Q_Z(Z_{t-1}, V_t) = \Phi Z_{t-1} + V_t$$

V_t serially independent $N(0, \Sigma)$ rv's, independent of e_t .

- e_t mutually independent Gaussian AR processes.

Model: macro state in US data



Objects of interest

- Persistence:

$$\rho(u_{it}, \eta_{i,t-1}, Z_t, Z_{t-1}) = \frac{\partial Q_\eta(\eta_{i,t-1}, Z_t, Z_{t-1}, u_{it})}{\partial \eta_{i,t-1}}.$$

- Exposure to aggregate shocks:

$$\beta(u_{it}, \eta_{i,t-1}, Z_t, Z_{t-1}) = \frac{\partial Q_\eta(\eta_{i,t-1}, Z_t, Z_{t-1}, u_{it})}{\partial Z_t}.$$

- Skewness:

$$\text{sk}(\eta, Z_t, Z_{t-1}) = \frac{Q_\eta(\eta, Z_t, Z_{t-1}, 0.9) + Q_\eta(\eta, Z_t, Z_{t-1}, 0.1) - 2Q_\eta(\eta, Z_t, Z_{t-1}, 0.5)}{Q_\eta(\eta, Z_t, Z_{t-1}, 0.9) - Q_\eta(\eta, Z_t, Z_{t-1}, 0.1)}.$$

- More: macro/micro IRFs + macro/micro risk quantification.

Framework

Identification & estimation

Identification: time series of panels

- The researcher observes:

$$W_t \quad t = 1, \dots, T, \quad \text{(time series of aggregates)}$$
$$Y_t \equiv \left\{ \left\{ y_{i,t+s} \right\}_{s=0}^{S-1} \right\}_{i \in \mathcal{I}_t} \quad t = 1, \dots, T, \quad \text{(time series of panels)}$$

where \mathcal{I}_t is a set of cross-sectional indexes of size N_t .

- S fixed to balance identification with credibly representative samples:
 - In between **repeated cross-sections** and **longitudinal panels**.
 - Precursor: Storesletten, Telmer, Yaron (2004).

Identification: large N , T with small S

- ▶ Assumptions

- **Macroeconometric step:**

$$W_t, t=1, \dots, T$$

$$\xRightarrow{T=\infty}$$

autocovariance function of $\{W_t\}$

→ $Q_Z(Z, V)$ is identified by factor model techniques.

- **Microeconometric step:**

$$\{y_{i,t+s}\}_{s=0}^{S-1}, i \in \mathcal{I}_t$$

$$\xRightarrow{N_t=\infty}$$

time- t cross-sectional CDF of $\{y_{i,t+s}\}_{s=0}^{S-1}$

→ $Q_{\eta,t}(\eta, u)$ is identified for each t by ABB techniques ($S \geq 4$).

Identification: large N , T with small S

- **Final step:** Pin down $Q_\eta(\eta, \tilde{Z}, Z, u)$ by linking $Q_{\eta,t}(\eta, u)$ to $\{Z_t\}$.
 - $Q_{\eta,t}(\eta, u)$ is equivalent to the time- t transition probability “matrix” $F_{\eta,t}(\tilde{\eta}|\eta)$.
 - Regressing $F_{\eta,t}(\tilde{\eta}|\eta)$ nonparametrically on $w_t^S = \{W_{t+s}\}_{s=0}^{S-1}$,

$$\underbrace{E \left[F_{\eta,t}(\tilde{\eta}|\eta) \mid w_t^S \right]}_{\text{known}} = \int \underbrace{F_\eta(\tilde{\eta}|\eta, \tilde{Z}, Z)}_{\rightarrow Q_\eta(\eta, \tilde{Z}, Z, u)} \underbrace{f_{Z|w}(\tilde{Z}, Z | w_t^S)}_{\text{known}} d(\tilde{Z}, Z).$$

- Under injectivity conditions, we can solve this equation for $Q_\eta(\eta, \tilde{Z}, Z, u)$.
- Key is the concurrence of time series of aggregates and panels:
 - Many short income histories subject to recessions/expansions over many cycles.

Data: time series of panels

- PSID:
 - Interviewed an initial sample representative of US households in 1968.
 - Thereafter, kept track of initial households and offspring + refresher/immigrant samples.
 - Interviews are annual between 1968 to 1997, biennial from 1999.
- We construct a **time series of panels** from PSID that coexists with the **aggregate data**:
 - Each panel has $S = 4$, made biennial for comparability (but we use all years).
 - $y = \log$ income net of education/family size/state of residence/time trends/etc.
 - **Male earnings**: labor income of representative person (age 25-to-60/male/married).
 - **Disposable income**: labor income of representative person and spouse + transfers - taxes.
 - We include age as additional argument in Q_η , Q_{init, t_0} but show results averaged over age.

Estimation: flexible parametric model

- Flexible specification of Q_η :

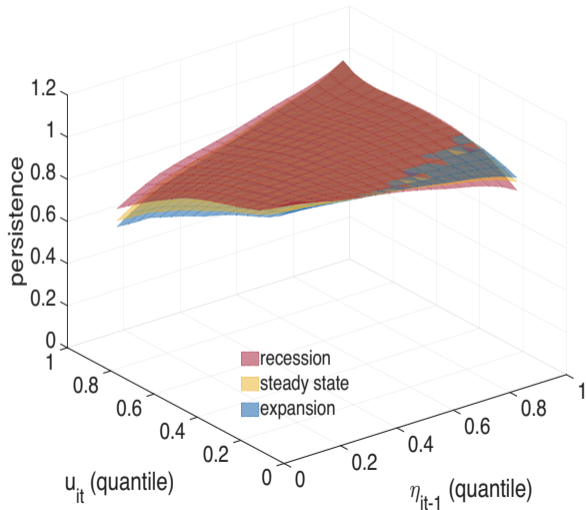
$$Q_\eta(\eta, Z_t, Z_{t-1}, u) = \psi(\eta)' \Theta(u) \varphi(Z_t, Z_{t-1}) = \sum_j \sum_k \psi_j(\eta) \theta_{jk}(u) \varphi_k(Z_t, Z_{t-1}),$$

- $\psi(\cdot)$, $\varphi(\cdot, \cdot)$: vectors of known basis functions (e.g., orthogonal polynomials).
 - $\Theta(\cdot)$: matrix of linear splines switching to exponential in the tails \implies parameters θ .
- Flexible time-varying specification of $Q_{\varepsilon,t}$ and Q_{init,t_0} .
 - We study the econometrics of this class of models:
 - Simulation-based estimation algorithm. ▶ Pseudo stochastic EM
 - Asymptotic approximations. ▶ Large-sample properties
 - Bootstrap inference accounting for cross-sectional + unit-level dependence. ▶ Bootstrap
 - ▶ Empirical specification and model fit

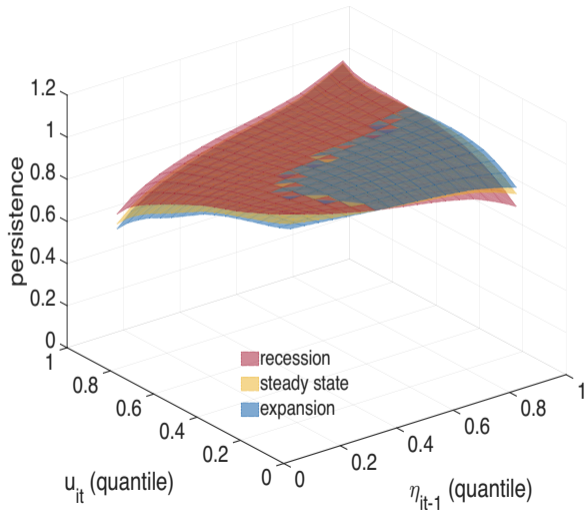
Macro shocks and nonlinear dynamics

Nonlinear persistence is macro and micro state-dependent: $\rho(u, \eta, Z_t, Z_{t-1})$

(a) Male earnings

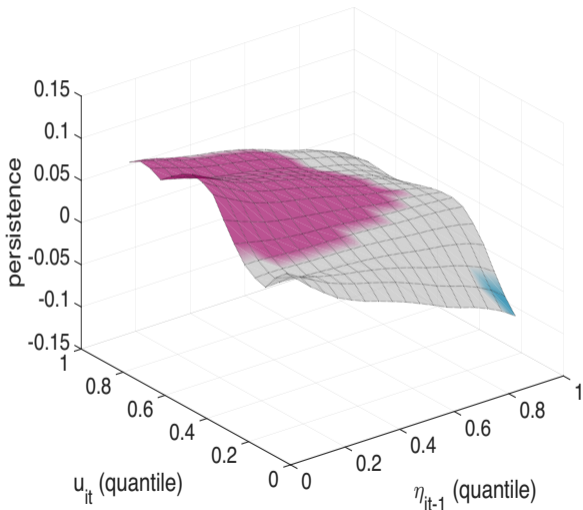


(b) Disposable income

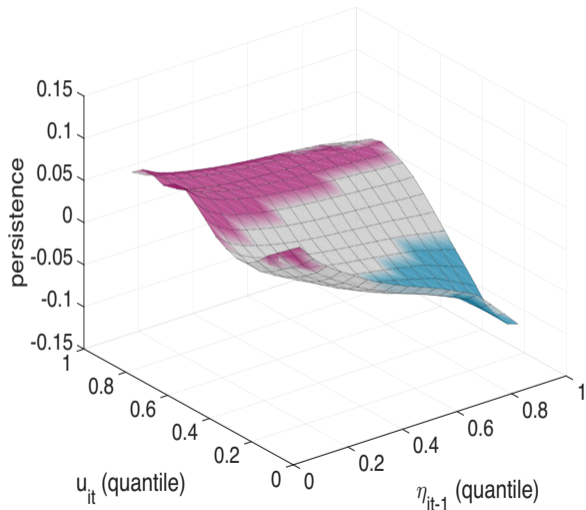


Differences in persistence between recessions and expansions

(a) Male earnings

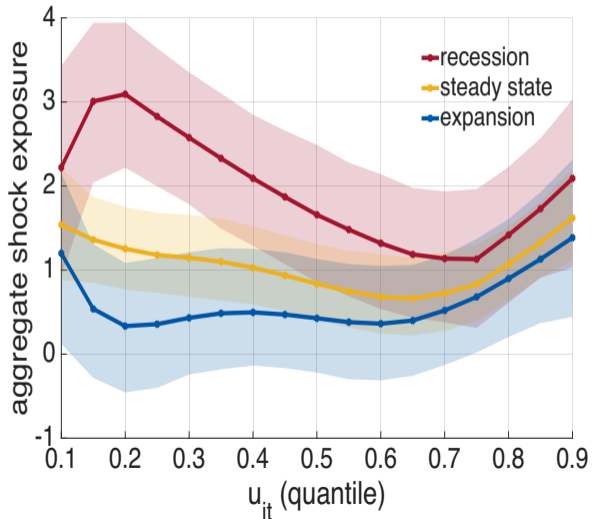


(b) Disposable income

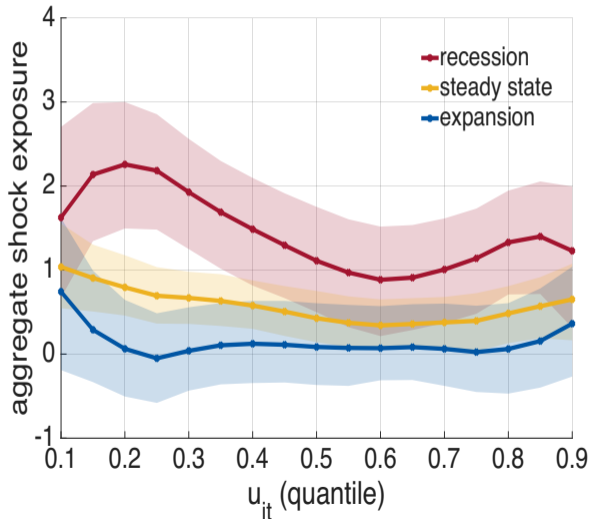


Exposures to aggregate shocks are countercyclical: $\beta(u, \eta, Z_t, Z_{t-1})$

(a) Male earnings

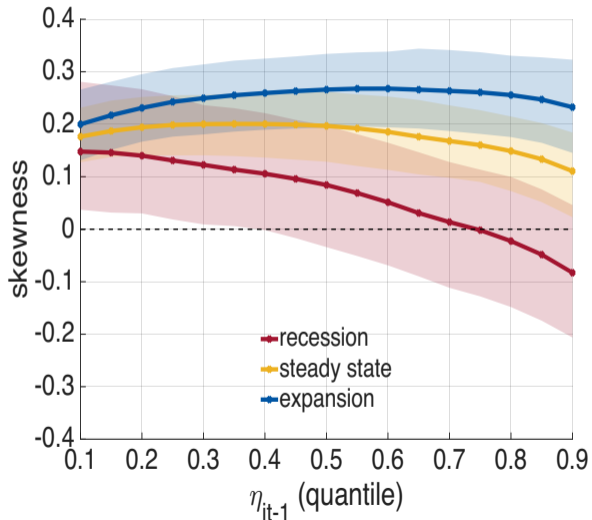


(b) Disposable income

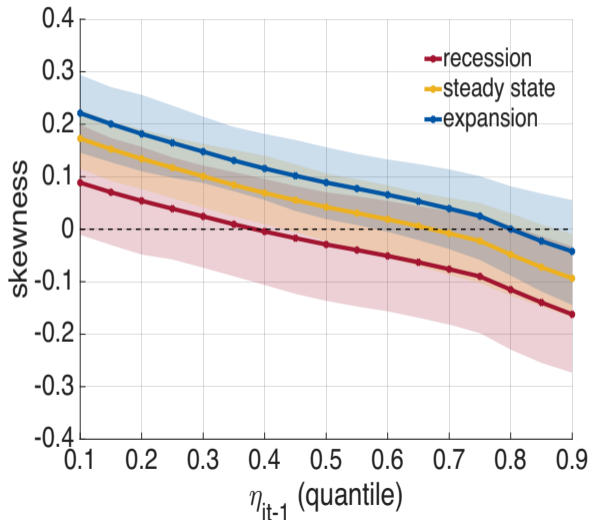


A tale of two skewnesses: $sk(\eta, Z_t, Z_{t-1})$

(a) Male earnings

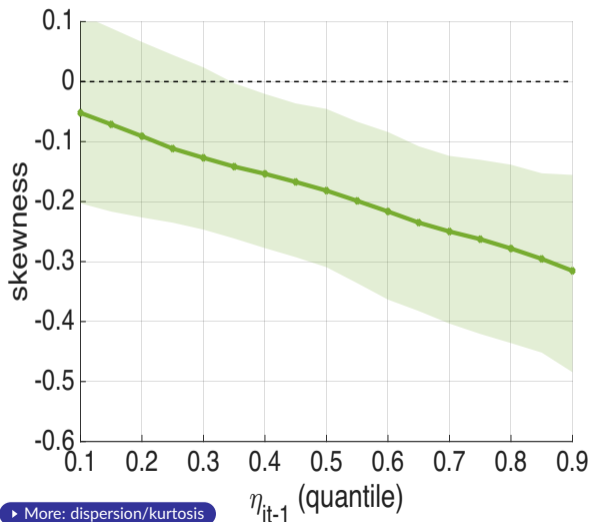


(b) Disposable income

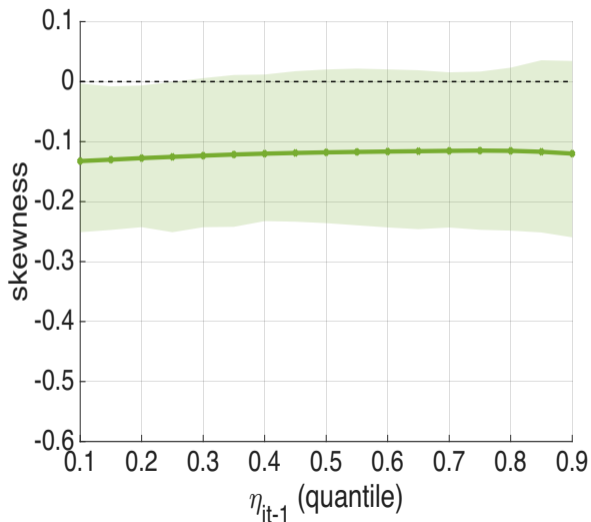


Differences in skewness between recessions and expansions

(a) Male earnings



(b) Disposable income



► More: dispersion/kurtosis

Macro/micro impulse responses

Nonlinear macro/micro IRFs

- We extend to our nonlinear macro/micro setup the idea in Gallant, Rossi, Tauchen (1993):
 - Fix initial state benchmark value, perturb it and track the evolution of outcomes.
- **Macro impulse responses.** Perturbation $g(Z_t^b) = g(Z_t^b + \Delta(\delta)) - \delta$:

$$\text{IRF}_{\eta Z}(h, \delta) = \frac{E\left[\eta_{i,t+h} \mid \eta_{i,t-1}, Z_t^b + \Delta(\delta), Z_{t-1}\right] - E\left[\eta_{i,t+h} \mid \eta_{i,t-1}, Z_t^b, Z_{t-1}\right]}{\delta}.$$

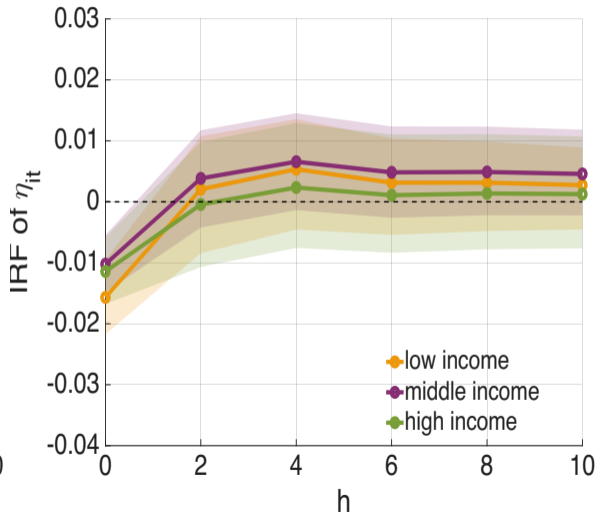
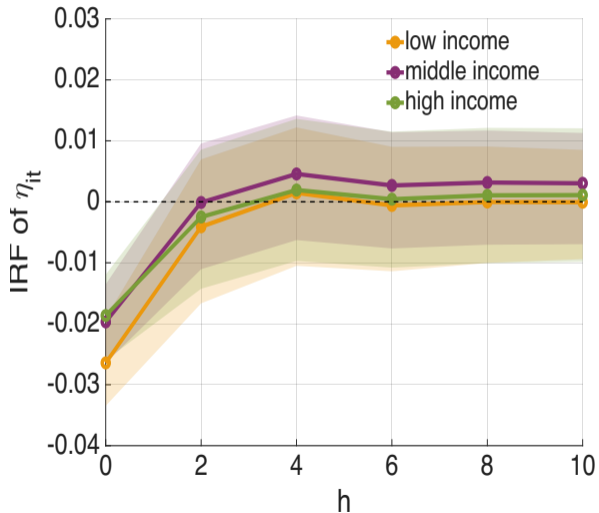
- **Micro impulse responses.** Perturbation $g(\eta_{i,t-1}^b) = g(\eta_{i,t-1}^b + \Delta(\delta)) - \delta$:

$$\text{IRF}_{\eta\eta}(h+1, \delta) = \frac{E\left[\eta_{i,t+h} \mid \eta_{i,t-1}^b + \Delta(\delta), Z_t, Z_{t-1}\right] - E\left[\eta_{i,t+h} \mid \eta_{i,t-1}^b, Z_t, Z_{t-1}\right]}{\delta}.$$

Responses to macro shocks are short-lived

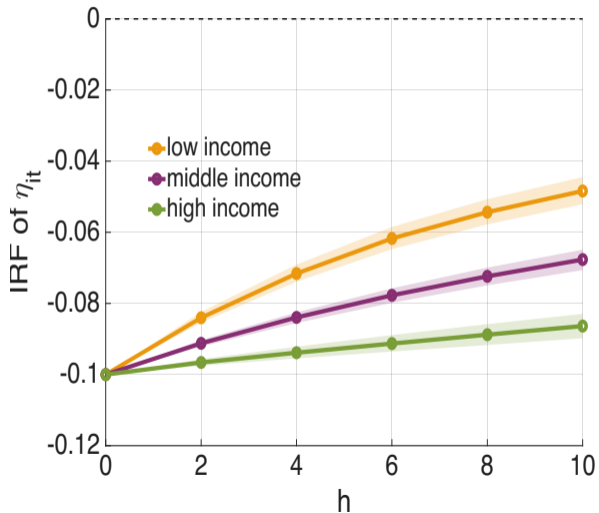
(a) Male earnings

(b) Disposable income

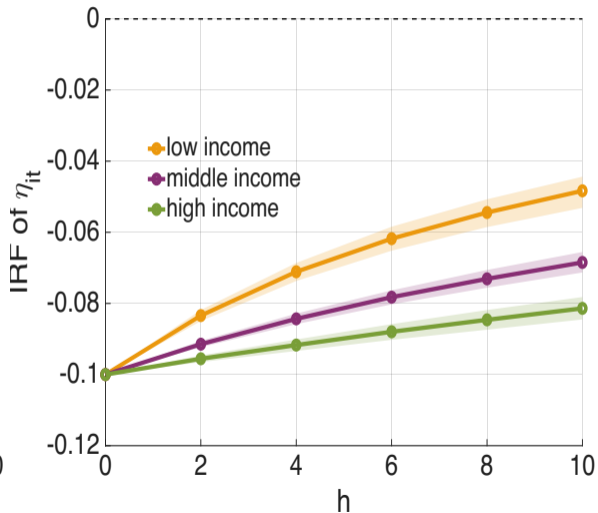


Responses to micro shocks decay slowly

(a) Male earnings



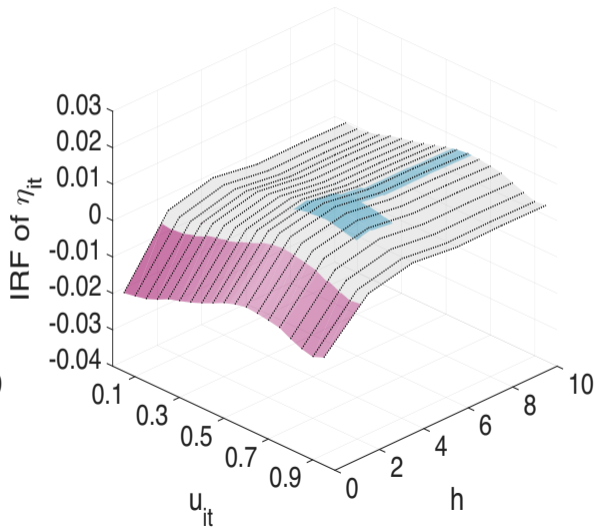
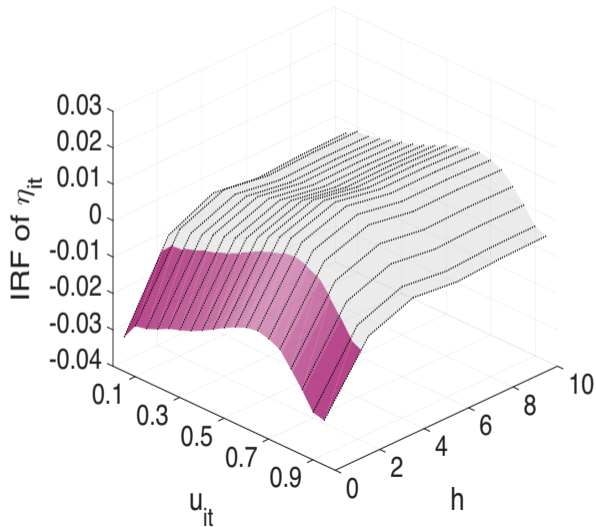
(b) Disposable income



Distributional responses to macro shocks are U-shaped

(a) Male earnings

(b) Disposable income



Macro/micro risk quantification

Cost of business cycle risk

- Cost of macro/micro sources of income risk: find CV such that

$$\mathbb{E} \left[\sum_{h=1}^H \delta^h U \left((1 - \text{CV}) e^{\eta_{i,t+h}} \right) \mid \text{no shocks}, \eta_{it}, Z_t \right] = \mathbb{E} \left[\sum_{h=1}^H \delta^h U \left(e^{\eta_{i,t+h}} \right) \mid \eta_{it}, Z_t \right].$$

Below we set $U(y) = y^{1-\gamma}/(1-\gamma)$ with $\gamma = 3$ and $\delta = (0.96)^2$.

- One focus of the literature is curvature in preferences:
 - Typically need high risk-aversion to obtain even minimal costs of fluctuations.
- **Extra channel.** Interaction between marginal utility and macro nonlinearities:
 - Countercyclical $\beta(u, \eta, Z_t, Z_{t-1})$ can generate large costs of risk!

Risk calculation: small-noise approximation

- Small-noise second-order Taylor expansion of compensating variation:
 - Given η_{it} , Z_t , write $\eta_{i,t+h} = \tilde{\eta}_{it}^h(u_{i,t+1}^h, V_{t+1}^h)$ with $u_{i,t+1}^h = (u_{i,t+\ell}^h)_{\ell=1}^h$, $V_{t+1}^h = (V_{t+\ell}^h)_{\ell=1}^h$.
 - Recenter u_{it} around zero and scale $u_{i,t+1}^h, V_{t+1}^h$ by ς_u, ς_V .
 - Curvature is determined by derivatives of $\tilde{U}(y) = U(e^y)$.
- Compensating variation for macro risk (for $\varsigma_u = 0$ and $\varsigma_V \rightarrow 0$):

$$CV_{\text{macro}} \approx -\frac{\sigma_V^2}{2} \frac{\sum_{h=1}^H \delta^h \sum_{\ell=1}^h \left(\tilde{U}''(\tilde{\eta}_{it}^h(0,0)) \left[\frac{\partial \tilde{\eta}_{it}^h(0,0)}{\partial V_{t+\ell}} \right]^2 + \tilde{U}'(\tilde{\eta}_{it}^h(0,0)) \left[\frac{\partial^2 \tilde{\eta}_{it}^h(0,0)}{\partial V_{t+\ell}^2} \right] \right)}{\sum_{h=1}^H \delta^h \tilde{U}'(\tilde{\eta}_{it}^h(0,0))}.$$

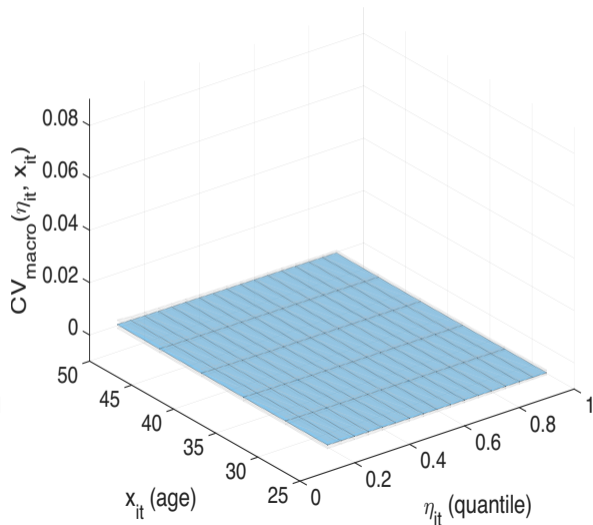
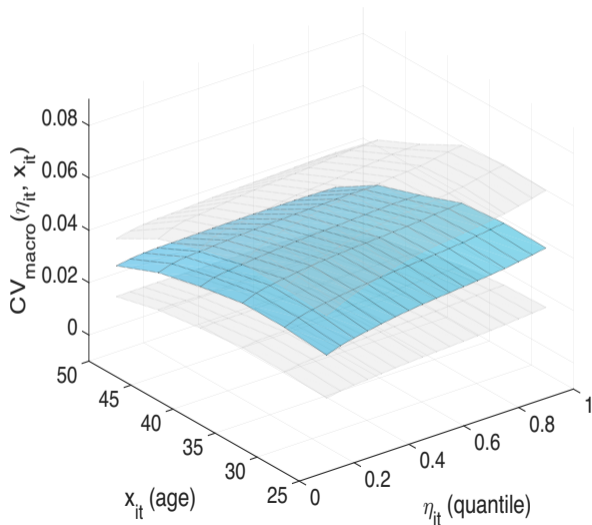
- Under log-utility, $\tilde{U}'(y) = 1$ and $\tilde{U}''(y) = 0$:

$$CV_{\text{macro}} \approx -\frac{\sigma_V^2}{2} \sum_{h=1}^H \underbrace{\frac{\delta^{h-1}(1-\delta)}{(1-\delta^H)}}_{>0, \text{ sum to } 1} \sum_{\ell=1}^h \underbrace{\left[\frac{\partial^2 \tilde{\eta}_{it}^h(0,0)}{\partial V_{t+\ell}^2} \right]}_{<0 \text{ for countercyclical } \beta}.$$

Nonlinear exposures amplify cost of macro risk

(a) Countercyclical $\beta(u, \eta, Z_t, Z_{t-1})$

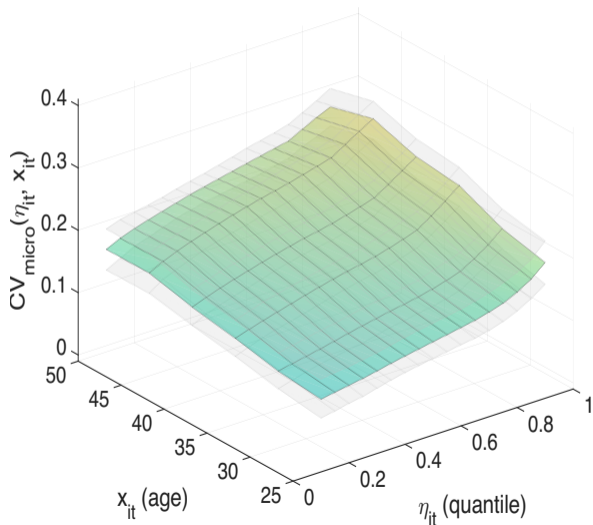
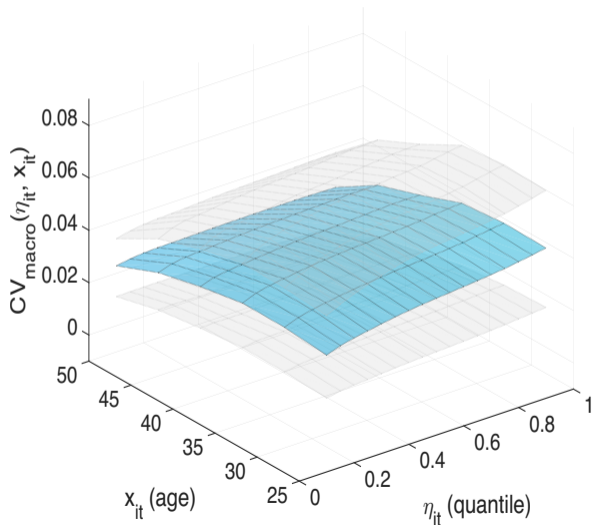
(b) Acyclical $\beta(u, \eta, Z_t, Z_{t-1})$



Macro risk is comparable to micro risk for young/low- η units

(a) Macro risk

(b) Micro risk



Conclusion

Conclusion

- Nonlinear framework to study transmission of aggregate/idiosyncratic shocks to income, leveraging macro and micro data and identification:
 - Our setup extends to multiple macro/micro state variables and richer feedback channels.
 - Tools to build reduced forms for heterogeneous agents models with macro shocks.
- We document aggregate state-dependence in persistence, skewness, exposures.
- Cyclicity in micro exposures to macro shocks matters for welfare calculations:
 - More general interest in measuring exposure of consumption and different forms of wealth to aggregate shocks \implies our framework offers an avenue to potentially do this.

Thank you!

Backup slides

Assumptions

1 Macro states:

- a $\{Z_t\}$ satisfies model in slide 8 with $V_t \stackrel{\text{iid}}{\sim} N(0, 1)$.
- b The entries of $\{e_t\}$ and omitted aggregate shocks $\{G_t\}$ are mutually independent and independent of $\{Z_t\}$, e_t is stationary, and G_t is i.i.d. over t .

2 Micro processes with macro states:

- a $\{\eta_{it}, \varepsilon_{it}\}$ i.i.d. across i given $\{Z_t, e_t, G_t\} \implies$ exchangeable given $\{Z_t, e_t\}$.
- b $\eta_{it}, \varepsilon_{it}$ satisfy model in slide 7 with $u_{it}, v_{it} \stackrel{\text{iid}}{\sim} U(0, 1)$ conditional on $\{Z_t\}$.

3 Atomicity:

- Micro shocks u_{it}, v_{it} are independent of macro shocks $\{V_t, e_t, G_t\}$.

Backup: pseudo stochastic EM algorithm

Notation: $\bar{Z} = \{Z_t\}_{t=0}^{T+S}$, $\bar{W} = \{W_t\}_{t=1}^{T+S}$, $\bar{y}_{it}^S = \{y_{i,t+s}\}_{s=0}^{S-1}$, $\bar{Z}_t^S = \{Z_{t+s}\}_{s=0}^{S-1}$.

Estimation algorithm

Initialize $\hat{\theta}^{(0)}$, $\{\hat{\delta}_t^{(0)}\}_{t=1}^T$. For $j = 1, \dots, J$, iterate between the following:

① Pseudo-Stochastic E step:

- i draw $\bar{Z}(j) = \{Z_t(j)\}_{t=0}^{T+S}$ from the macro posterior $f(\bar{Z} | \bar{W}, \hat{\lambda})$,
- ii independently over units i and subpanels t , draw $\bar{\eta}_{it}^S(j) = \{\eta_{i,t+s}(j)\}_{s=0}^{S-1}$ from the micro posterior $f(\bar{\eta}_{it}^S | \bar{y}_{it}^S, \bar{Z}^S(j), \hat{\theta}^{(j-1)}, \hat{\delta}_t^{(j-1)})$.

② Pseudo M step:

- i update parameters to $\hat{\theta}^{(j)}$ and $\{\hat{\delta}_t^{(j)}\}_{t=1}^T$ by quantile and exponential regressions treating $\left\{ \left\{ \{\eta_{i,t+s}(j), y_{i,t+s}, x_{i,t+s}, Z_{t+s}(j)\}_{s=0}^{S-1} \}_{i \in \mathcal{I}_t} \right\}_{t=1}^T$ as data.

For some $\mu \in (0, 1)$, set $\hat{\theta} = (\mu J)^{-1} \sum_{j=(1-\mu)J}^J \hat{\theta}^{(j)}$ and $\hat{\delta}_t = \sum_{j=(1-\mu)J}^J \hat{\delta}_t^{(j)}$.

Backup: large-sample properties

- Plug-in estimator of summaries: $\hat{\gamma} = \gamma(\hat{\theta})$ for some smooth function γ .
- Estimator $\hat{\theta}$ sets to zero the sample counterpart to **doubly-integrated moments**:

$$E \left[\int \left(\frac{1}{N_t} \sum_{i \in \mathcal{I}_t} \int m_{\theta}(\theta_0; \bar{y}_{it}^S, \bar{\eta}^S, \bar{Z}_t^S) f(\bar{\eta}^S | \bar{y}_{it}^S, \bar{Z}_t^S, \theta_0, \delta_t) d\bar{\eta}^S \right) f(\bar{Z}^S | \bar{W}, \lambda_0) d\bar{Z}^S \right] = 0$$

where m_{θ} collects quantile/exponential regression orthogonality conditions.

- Taylor expansions of moment conditions around $(\theta_0, \lambda_0) \implies$ asymptotic distribution.

Asymptotic approximations

As $N, T \rightarrow \infty$, $\sqrt{T}(\hat{\theta} - \theta_0) \xrightarrow{d} N(0, \Sigma_{\theta_0})$ and $\sqrt{T}(\hat{\gamma} - \gamma_0) \xrightarrow{d} N(0, J_{\theta_0} \Sigma_{\theta_0} J'_{\theta_0})$ with Σ_{θ_0} symmetric, positive semi-definite and J_{θ_0} the Jacobian of γ evaluated at θ_0 .

Backup: bootstrap

- Omitted aggregate factors:

- Common shocks $G_t = (G_{\eta,t}, G_{\varepsilon,t}, G_{\text{init},t}) \stackrel{\text{iid}}{\sim} U(0, 1)$ drive cross-sectional dependence.
- Micro ranks generated via Gaussian copula:

$$u_{it} = \Phi \left(c_{\eta} \Phi^{-1}(G_{\eta,t}) + \sqrt{1 - c_{\eta}^2} \Phi^{-1}(\tilde{u}_{it}) \right), \text{ similarly for } v_{it}, \nu_{i,t_0}.$$

- Parameters $c_{\eta}, c_{\varepsilon}, c_{\text{init}}$ estimated by sample mean of copula-inverted ranks over EM draws.

- Unit overlap:

- For individuals in consecutive odd/even panels, idiosyncratic ranks follow

$$\left(\Phi^{-1}(\tilde{u}_{it}) \quad \Phi^{-1}(\tilde{u}_{it'}) \right)' \sim N \left(0, \begin{pmatrix} 1 & d_{\eta} \\ d_{\eta} & 1 \end{pmatrix} \right), \text{ similarly for } \tilde{v}_{it}, \tilde{\nu}_{i,t_0}.$$

- Parameters $d_{\eta}, d_{\varepsilon}, d_{\text{init}}$ estimated alongside $c_{\eta}, c_{\varepsilon}, c_{\text{init}}$ within EM.

Backup: bootstrap

Bootstrap approach

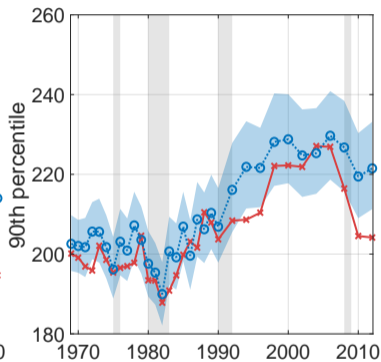
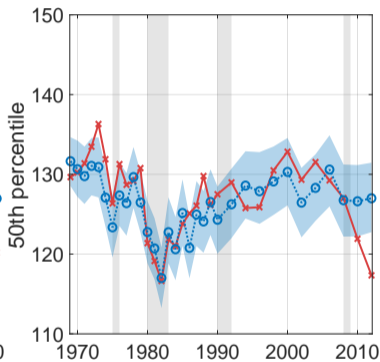
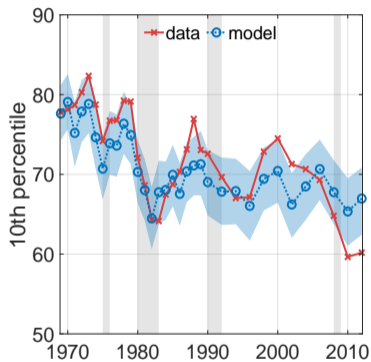
Given estimates of $(c_\eta, c_\varepsilon, c_{\text{init}}, d_\eta, d_\varepsilon, d_{\text{init}})$, do the following:

- ① Simulate the time series of aggregate factors $\{G_{\eta,t}, G_{\varepsilon,t}, G_{\text{init},t}\}_{t=1}^T$.
- ② For each unit i determine the first (t_0) and last (t_1) period in the dataset. Next,
 - i draw the path of idiosyncratic shocks $\{\tilde{u}_{it}, \tilde{v}_{it}, \tilde{\nu}_{it}\}_{t_0 \leq t \leq t_1}$ imposing the correlations d_η , d_ε and d_{init} across consecutive periods;
 - ii combine aggregate and idiosyncratic factors to obtain $\{u_{it}, v_{it}, \nu_{it}\}_{t_0 \leq t \leq t_1}$ imposing the cross-sectional dependence implied by c_η , c_ε and c_{init} ;
 - iii for the first two periods, use $Q_{\text{init},t}$ and ν_{it} to generate η_{it} ;
 - iv for every other period, use Q_η and u_{it} to generate η_{it} ;
 - v for all periods, use $Q_{\varepsilon,t}$ and v_{it} to generate ε_{it} ;
 - vi form $y_{it} = \eta_{it} + \varepsilon_{it}$ for all $t_0 \leq t \leq t_1$.
- ③ Assign the data to the appropriate unit and time cell.

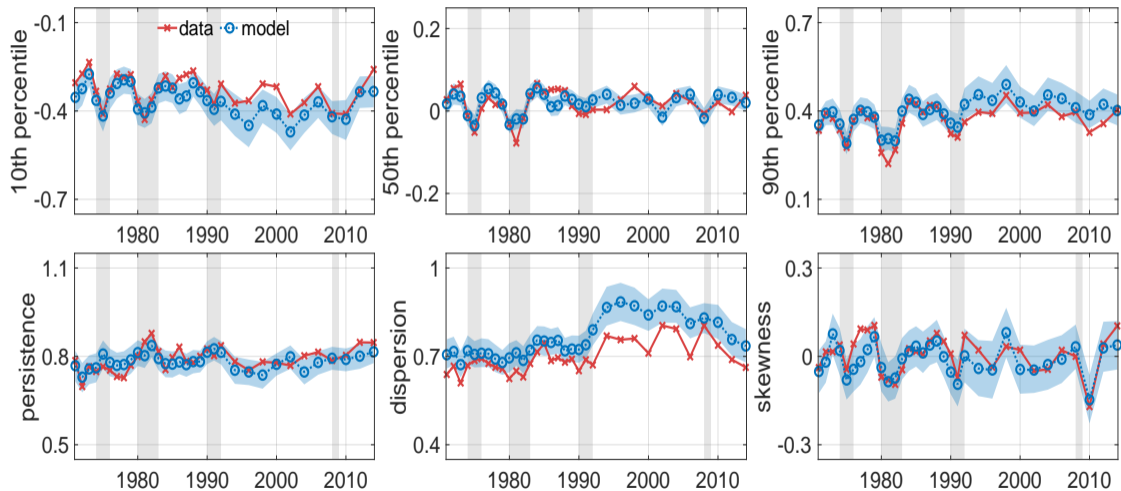
Backup: empirical specification

- Specification of Q_η :
 - ψ is third-order Hermite polyn on $\eta \times$ second-order Hermite polyn on age x .
 - φ is second-order Hermite polyn on (Z_t, Z_{t-1}) but with restrictions:
 - linear term excluded from (η, x) interactions, quadratic term only in the intercept.
 - Grid on rank space $L = 11$.
- Specification of $Q_{\varepsilon,t}$:
 - Time effects + $L = 11$.
- Specification of Q_{init,t_0} :
 - Time effects + second-order Hermite polyn on age x + $L = 11$.

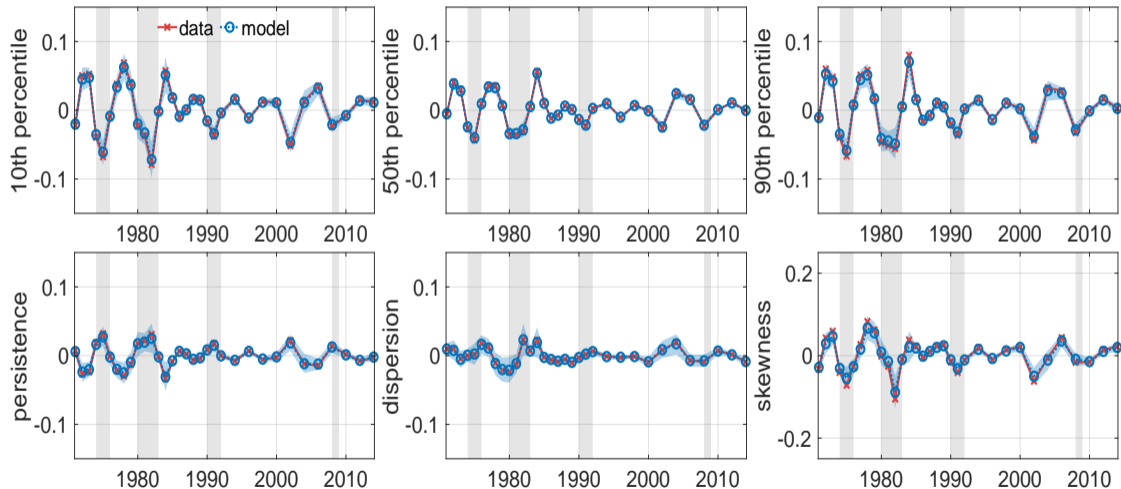
Backup: model fit (income level, thousands of 2016 US\$)



Backup: model fit (biennial income growth)



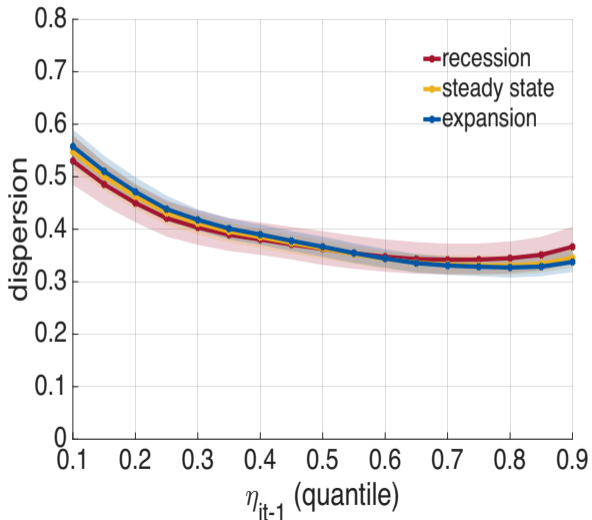
Backup: model fit (biennial income growth, projection on aggregate state)



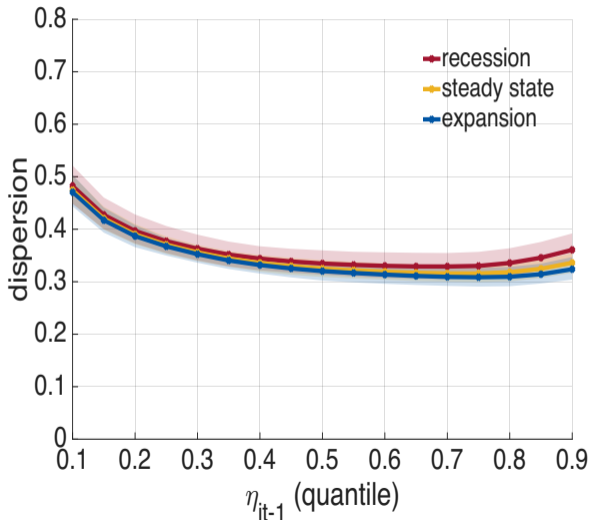
▶ Back

Backup: dispersion

(a) Male earnings

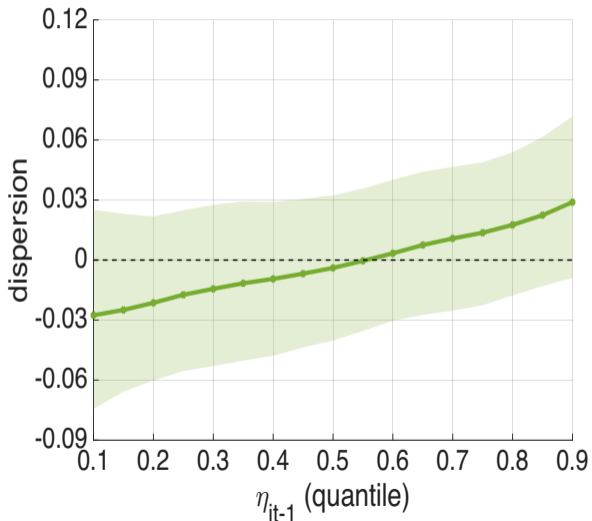


(b) Disposable income

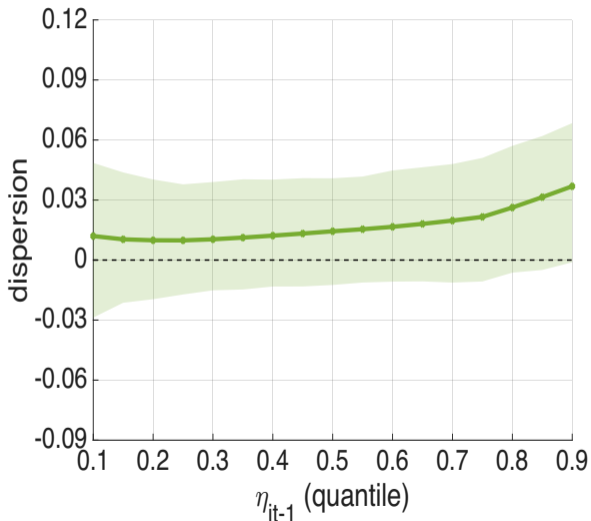


Backup: dispersion (difference between recessions and expansions)

(a) Male earnings

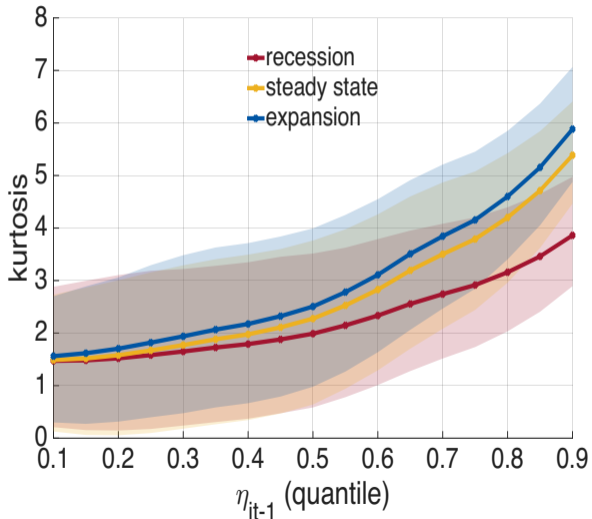


(b) Disposable income

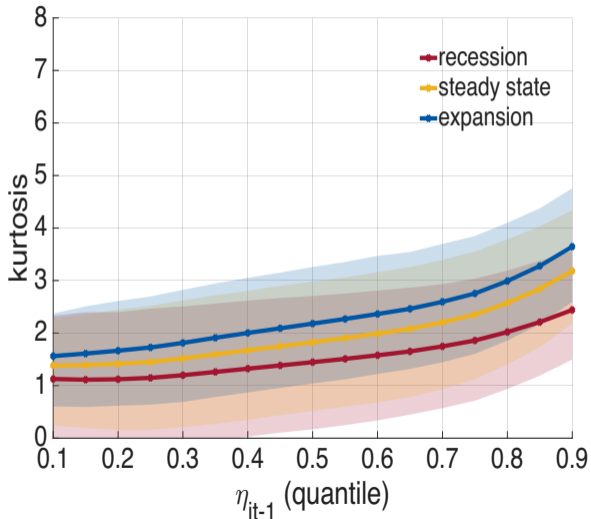


Backup: kurtosis

(a) Male earnings

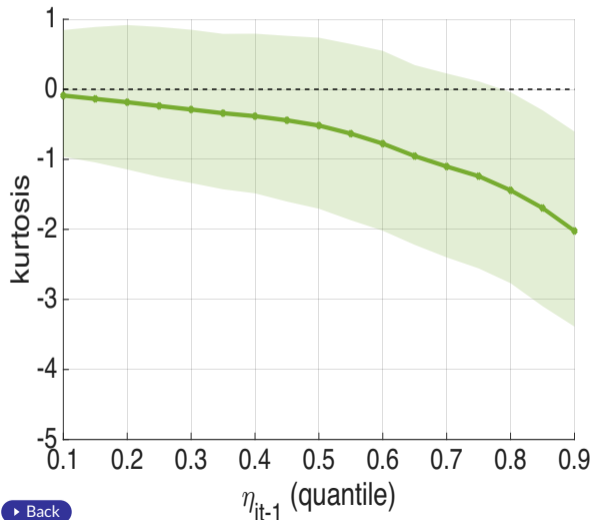


(b) Disposable income



Backup: kurtosis (difference between recessions and expansions)

(a) Male earnings



(b) Disposable income

